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Abstract

A new method for the design of waverider configurations with generalized shock geometries is presented. An arbitrary three-dimensional shock shape is specified as input, and a cross-stream Euler marching procedure is used to define the post-shock flowfield. Unlike most previous studies, this approach allows for the use of nonaxisymmetric shock topologies with nonconstant shock strengths. The problem's fundamental ill-posedness is suppressed by reformulating the problem in the proper curvilinear coordinate system. Details of the new design approach are given, and the method is validated using comparisons with exact theory and direct numerical simulations.

Nomenclature

a/b	= ratio of vert. and horiz. axis of ellipse
L/D	= lift over drag
M_∞	= freestream Mach number
OP	= osculating plane
p	= pressure
T	= temperature
u, v, w	= vel. components in Cartesian coords.
U, V, W	= vel. components in generalized coords.
x, y, z	= Cartesian coordinates
β	= local shock angle
δ	= displacement thickness
γ	= ratio of specific heats
ρ	= density
θ_w	= two-dimensional wedge angle
ξ	= streamwise computational coordinate
η	= circumferential computational coord.
ζ	= radial computational coordinate

Introduction

When one examines humankind's past a clear trend has prevailed throughout history, and that is the everpresent desire to travel greater distances in shorter times. Indeed, this desire has been a primary motivation for technological growth and expansion of the scientific community. For some the interest lies in fulfilling the transportation needs for a so-called 'Global Community' as suggested by Kuchemann[1], where any two places on the globe are at most a few hours of travel time apart. For others the interest lies in providing more efficient access to Low Earth Orbit (LEO) with fully reusable aircraft that take off and land on conventional runways. A number of projects have been initiated over the past decade or so to research these and related topics. These projects include the design of such vehicles as the National AeroSpace Plane (NASP), the German Sänger, High-Speed Civil Transports (HSCTs), and a host of other aircraft. Recent advances in the fields of materials and propulsion have made vehicles such as these realistically feasible for the early 21st century.

In the quest for high-speed performance an old idea, the waverider, has resurfaced as a viable class of geometries. Waveriders, classically defined in inviscid flow, have sharp leading edges and maintain attached shock waves at the design flight conditions. The shock wave (or shock waves) is generally contained beneath the body in such a way that the aircraft appears to be riding on the wave, hence the name waverider.

Waveriders are interesting vehicles for several reasons. From a performance standpoint they have both theoretically and experimentally provided high values of L/D at high Mach numbers at their on-design flight conditions. Additionally, recent experimental work by Bauer et al.[2] has shown them to be competitive at off-design conditions as well. From a design standpoint waveriders offer the rather unique feature of isolating the flows over the upper and lower surfaces, effectively dividing their design into two independent problems. The lower surface is designed to generate a desired shock wave and inlet flow conditions, and the upper surface may be independently designed to fulfill both performance and internal volume requirements.

Waveriders were first conceptualized in 1959 by Nonweiler[3] as reentry vehicles for manned space flight. Nonweiler's classic 'caret-wing' produced a planar shock wave contained beneath a delta planformed aircraft as illustrated in fig. 1. The caret-wing was 'carved' from the inviscid, two-dimensional, supersonic flow over an infinite wedge. By taking the known flowfield and

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defining a leading edge on the shock surface (an inverted-V shape in this case) the streamsurface passing through the leading edge could be used to define the waverider's lower surface. The upper surface was then chosen as parallel to the freestream flow, thus defining a body with internal volume. The post-shock pressure on the lower surface was higher then the freestream pressure on the upper-surface providing the aircraft's lift.

A similar approach was used in 1963 by Jones[4] to carve waverider topologies from the flowfields over axisymmetric cones at zero incidence. In the following years many researchers (see Eggers[5] for a summary) provided variations to this design approach with the primary difference being the choice of the flowfield from which the waverider is carved. In most past studies geometrically simple shock shapes such as planes or axisymmetric surfaces were chosen because exact or relatively simple approximate solutions for the post-shock flowfields were known.

The design of waveriders is inherently an inverse problem. In Nonweiler's design approach, the nature of the flowfield was known prior to the shape of the waverider; thus the configuration was inversely designed to reproduce a desired shock and flowfield. It is actually quite difficult to design a vehicle that will perform as a waverider at some desired cruise conditions using direct methods. However, it is a relatively simple task, in theory if not in practice, to choose a desired shock wave and find the waverider that will generate it. Inverse methods are often superior to direct methods for use in optimization procedures as well. Past work by Center et al.[6] has shown that waveriders can be designed and viscous analysis performed in a few seconds on a workstation using inverse techniques, whereas a direct Navier-Stokes simulation of the same configuration may require more than 30 minutes of Cray YMP CPU time.

The current study again employs an inverse design method with the same basic design approach previously outlined. However, where most past studies have limited the choice of shock shapes to planar or axisymmetric surfaces, the current effort allows for the use of more general shock shapes than those of any previous studies. The method incorporates a cross-stream Euler marching scheme that defines the flowfield behind the specified shock surface from which the waverider is carved. The general three-dimensional, supersonic, cross-stream marching problem is ill-posed; however, if correctly formulated the method can be solved to yield accurate and physically meaningful solutions.

Details of the marching scheme and peripheral computations are given here, with comparisons to exact theory and direct Euler simulations.

Design Approach

In the introduction a very brief summary of the basic waverider design approach was given. In this section a more elaborate description will be given with details pertaining to the current study.

As was previously mentioned the upper and lower surfaces may be designed independently, and the first step in the design of the lower surface is the specification of the desired shock geometry and leading edge shape. The post-shock conditions which are the actual initial conditions for the marching procedure are computed next. The solution is then marched away from the shock surface in 'optimal' directions. Once the flowfield is defined, streamlines are integrated downstream from the specified leading edge to define the streamsurface that becomes the waverider's lower surface. The upper surface is arbitrarily defined by the user, and flow parameters are computed there using an approximate method of characteristics. Performance of the vehicle is computed by integrating the surface pressure over the surface. Details of the primary steps are given in the following subsections.

Lower Surface Design

The lower (shock generating) surface is the most difficult surface to design, but it is really the central issue of the problem, since it typically generates about 80% of a waverider's total lift. It is of particular interest here, as it demonstrates the utility of a newly developed cross-stream, Euler marching scheme.

Initial Conditions The first step in the design of the lower surface is the definition of the shock surface. Ideally the shock is parametrically defined, such that the computational grid on the shock surface can be optimally generated. As will become evident in a later section, the lines of constant η value should be aligned with the local osculating plane (OP), where the OP at a point is tangent to the local streamline and contains the principal normal to the streamline as illustrated in fig. 2. The OP is the plane in which a general three-dimensional flow most closely resembles an axisymmetric flow. Creating a computational surface mesh of this nature on the parametrically defined shock requires the numerical integration of a single formula obtained through vector analysis. The geometric orientation of the shock wave and the computational mesh are portrayed in fig. 3.

Once the shock surface and the freestream conditions M_∞ and γ are specified the post-shock flow conditions can be defined. The Rankine-Hugoniot jump relations are used to compute the post-shock scalar quantities, and several simple geometric relations are used to compute the post-shock velocity components, u, v, and w.

Governing Equations The five equations governing the flowfields of this study are the conservation of mass, the three conservation of momentum, and the entropy equations for steady, inviscid, three-dimensional flow, given in vector notation by

$$\mathbf{E}\vec{q}_x + \mathbf{F}\vec{q}_y + \mathbf{G}\vec{q}_z = 0 \tag{1}$$

where

$$\mathbf{E} = \begin{bmatrix} u & \rho & 0 & 0 & 0 \\ 0 & \rho u & 0 & 0 & 1 \\ 0 & 0 & \rho u & 0 & 1 \\ 0 & 0 & 0 & \rho u & 1 \\ -up\gamma & 0 & 0 & 0 & \rho u \end{bmatrix},$$

$$\mathbf{F} = \begin{bmatrix} v & 0 & \rho & 0 & 0 \\ 0 & \rho v & 0 & 0 & 1 \\ 0 & 0 & \rho v & 0 & 1 \\ 0 & 0 & 0 & \rho v & 1 \\ -vp\gamma & 0 & 0 & 0 & \rho v \end{bmatrix},$$

$$\mathbf{G} = \begin{bmatrix} w & 0 & 0 & \rho & 0 \\ 0 & \rho w & 0 & 0 & 1 \\ 0 & 0 & \rho w & 0 & 1 \\ 0 & 0 & 0 & \rho w & 1 \\ -wp\gamma & 0 & 0 & 0 & \rho w \end{bmatrix},$$

and

$$\vec{q} = \begin{bmatrix} \rho \\ u \\ v \\ w \\ p \end{bmatrix}.$$

The subscripts denote partial derivatives. The use of the entropy equation in place of the energy equation is valid for inviscid, adiabatic flows, and the substitution of p/ρ^γ in place of the entropy is valid for calorically perfect gases. Since the problem at hand is an inviscid shock-fitting algorithm, these assumptions are valid and the use of the entropy equation is acceptable, in fact preferable, as it weakens the coupling between the system of equations. Note that the entropy equation does not require the entropy to be constant everywhere; it merely states that the entropy of a given particle is constant. Hence, the entropy is constant along streamlines in a steady flow.

Equation 1 is nondimensionalized as follows

$$\tilde{\rho} = \frac{\rho}{\rho_\infty},\ \tilde{u} = \frac{u}{a_\infty},\ \tilde{v} = \frac{v}{a_\infty},\ \tilde{w} = \frac{w}{a_\infty},$$

$$\text{and } \tilde{p} = \frac{p}{p_\infty \gamma}. \tag{2}$$

When these relations are substituted into eqn. 1 the system remains unchanged in form, so it is not repeated here, and the tildes are dropped throughout the remainder of the development.

The system of equations is transformed into a generalized coordinate system where $\xi = \xi(x,y,z)$, $\eta = \eta(x,y,z)$, and $\zeta = \zeta(x,y,z)$. Expanding the partial derivatives using the chain rule and simplifying where possible, yields the system

$$\hat{\mathbf{E}}\vec{q}_\xi + \hat{\mathbf{F}}\vec{q}_\eta + \hat{\mathbf{G}}\vec{q}_\zeta = 0 \tag{3}$$

where

$$\hat{\mathbf{E}} = \begin{bmatrix} U & \rho\xi_x & \rho\xi_y & \rho\xi_z & 0 \\ 0 & \rho U & 0 & 0 & \xi_x \\ 0 & 0 & \rho U & 0 & \xi_y \\ 0 & 0 & 0 & \rho U & \xi_z \\ -Up\gamma & 0 & 0 & 0 & \rho U \end{bmatrix},$$

$$\hat{\mathbf{F}} = \begin{bmatrix} V & \rho\eta_x & \rho\eta_y & \rho\eta_z & 0 \\ 0 & \rho V & 0 & 0 & \eta_x \\ 0 & 0 & \rho V & 0 & \eta_y \\ 0 & 0 & 0 & \rho V & \eta_z \\ -Vp\gamma & 0 & 0 & 0 & \rho V \end{bmatrix},$$

and

$$\hat{\mathbf{G}} = \begin{bmatrix} W & \rho\zeta_x & \rho\zeta_y & \rho\zeta_z & 0 \\ 0 & \rho W & 0 & 0 & \zeta_x \\ 0 & 0 & \rho W & 0 & \zeta_y \\ 0 & 0 & 0 & \rho W & \zeta_z \\ -Wp\gamma & 0 & 0 & 0 & \rho W \end{bmatrix},$$

where U, V, and W are the contravariant velocities and ξ_x, ξ_y, ξ_z, η_x, η_y, η_z, ζ_x, ζ_y, and ζ_z are the inverse metrics.

As will be shown in a later section, marching within the OP minimizes the effects of the problem's ill-posedness. In the OP the contravariant velocity V is by definition exactly zero. Applying this to eqn. 3 yields

$$\hat{\mathbf{E}}\vec{q}_\xi + \hat{\mathbf{F}}^*\vec{q}_\eta + \hat{\mathbf{G}}\vec{q}_\zeta = 0 \tag{4}$$

where

$$\hat{\mathbf{F}}^* = \begin{bmatrix} 0 & \rho\eta_x & \rho\eta_y & \rho\eta_z & 0 \\ 0 & 0 & 0 & 0 & \eta_x \\ 0 & 0 & 0 & 0 & \eta_y \\ 0 & 0 & 0 & 0 & \eta_z \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} .$$

This is the system of equations that is used to describe the flowfield between the shock and the waverider's lower surface.

Ill-posedness For supersonic flow problems marching is usually performed in the streamwise or time-like direction as illustrated in fig. 4a. In this problem, however, availability of initial data mandates a cross-stream marching direction as shown in fig. 4b.

It was previously stated that the general three-dimensional, supersonic, cross-stream marching problem is ill-posed or, put more simply, the wrong boundary conditions are given for the set of governing equations. This can be seen graphically in fig. 5 where the initial data clearly lies outside the Mach conoids defining the domains of influence and dependence. This can be illustrated mathematically by looking at the linear model equation

$$\phi_{xx} - \phi_{yy} - \phi_{zz} = 0. \tag{5}$$

This is a hyperbolic equation (as is eqn. 4 for supersonic flow) with x as the time-like direction. A solution to eqn. 5 can be constructed from an infinite series of modal components of the form

$$\phi = \exp[i(k_1x + k_2y + k_3z)], \tag{6}$$

where $i = \sqrt{-1}$ and k_1, k_2, and k_3 are the wavenumbers in the x, y, and z directions, respectively. Cross-stream marching implies marching in a direction different from the time-like direction. In the model problem the z-direction is used.

By differentiating eqn. 6 and substituting the results into eqn. 5, the relation

$$k_3 = \pm\sqrt{k_1^2 - k_2^2} \tag{7}$$

can be formed for k_3 as a function of k_1 and k_2. Equation 7 states that for $|k_2| > |k_1|$ the wavenumber k_3 becomes imaginary causing ϕ to grow exponentially in the marching direction.

On the other hand, it can easily be shown that a similar problem formulated in two dimensions is well-posed for either streamwise or cross-stream marching directions. Consequently, if a system of governing equations can be reduced to a two-dimensional system by the proper choice of marching directions then the ill-posedness can sometimes be removed.

A more appropriate model for the current system of equations is the two-dimensional, compressible streamfunction

$$A\psi_{xx} + 2B\psi_{xz} + C\psi_{zz} = D \tag{8}$$

where

$$A = 1 - \frac{u^2}{a^2}\ ,\ B = -\frac{uw}{a^2}\ ,\ C = 1 - \frac{w^2}{a^2}\ ,$$

$$\text{and}\quad D = \rho^2 T(M^2 - 1)F'(\psi)\ . \tag{9}$$

Here $F(\psi)$ is the entropy expressed in terms of the streamfunction. For irrotational flow $F'(\psi) = 0$, and eqn. 8 is homogeneous. A series solution of the form

$$\psi = \exp[i(k_1x + k_2y)] \tag{10}$$

can be constructed for the homogeneous equation, and the wavenumber in the z-direction is given by

$$k_2 = \frac{k_1}{1 - \frac{w^2}{a^2}}\left[\frac{uw}{a^2} \pm \sqrt{M^2 - 1}\right]\ . \tag{11}$$

Note that k_2 is real-valued for all $M > 1$, resulting in bounded solutions for supersonic flow.

If the flow is rotational (i.e., behind a curved shock), then $F'(\psi)$ is not zero, and eqn. 8 is not homogeneous. There is no longer a simple solution construction, and it is difficult to establish whether or not the solution will be bounded. However, if the radius of curvature of the shock is large with respect to the marching distance, then $F'(\psi)$ will be small, and the right hand side of eqn. 8 can be neglected.

By marching in the OP the contravariant velocity V is eliminated. Unfortunately, the system is not quite two-dimensional, as several derivative terms still exist in the matrix $\hat{\mathbf{F}}^*$. However, if these remaining terms are kept small in relation to the associated terms in the matrix $\hat{\mathbf{G}}$ (i.e., cross-stream gradients are small over distances comparable to the marching distance), then bounded solutions may be expected.

Grid Generation Marching is to be performed in the local OP; therefore, it should be obvious that the marching directions, and hence the computational grid, are solution-dependent. At the shock and at each marched grid layer the orientation of the local OPs are determined, indicating the directions in which the marching is to be performed. The marching distance must be limited within the characteristic boundaries of the given initial data. Figure 6 shows a typical symmetry-plane grid for a conical shock wave. Note that a grid point is lost at the upstream and downstream boundaries at each marched layer. This provides a means for implicitly defining boundary conditions, since initial data at these boundaries is not known explicitly.

Marching On the shock wave and at each marched grid layer the gradients of $\vec{q}$ with respect to ξ and η are computed using central finite differences. The gradients in the ζ direction are computed by rewriting eqn. 4 in the form;

$$\vec{q}_{\zeta} = -\hat{\mathbf{G}}^{-1}[\hat{\mathbf{E}}\vec{q}_{\xi} + \hat{\mathbf{F}}^{*}\vec{q}_{\eta}]. \tag{12}$$

Due to the sparseness of matrix $\hat{\mathbf{G}}$ its inversion is a relatively simple task. The solution is marched using a centrally differenced scheme, except at the shock surface where one-sided differences must be used.

Surface Definition The waverider's lower surface is a streamsurface in the inviscid flow, and it is defined by integrating streamlines downstream from the prescribed leading edge. The integration is performed two-dimensionally in the computational domain where the contravariant velocity, V, is identically zero. Figures 7a and 7b portray the computed streamlines and the waverider's surface, respectively.

Upper Surface Design

The design of the upper surface of the waverider is a direct design problem, and it offers more flexibility to the designer. The surface may be tailored to include features for internal volume placement and expansion surfaces for enhanced performance.

The upper surface typically provides less than 20% of the vehicle's lift even when optimally expanded. Due to this and because the design of the lower surface was the primary goal of the project, a much simpler (and correspondingly less accurate) solution method is applied to the upper surface. An approximate variation of the method of characteristics developed by Center et al.[6] is applied along pseudo-streamlines defining the upper surface.

Performance Analysis

The vehicle's performance is computed by integrating the surface pressure over the waverider's surface. Integration of the pressure forces is performed by computing the area vector at each grid cell and multiplying the Cartesian area components by the cell-centered pressure coefficient.

Validation

As with any new computational method, an extensive validation process is needed to test the accuracy and the limits of the new method. This is done through comparisons with exact theory and other numerically obtained results.

Exact Solutions

Several exact solutions exist with which the inviscid design of the lower surface may be validated. The simplest case is the two-dimensional flow over a wedge with a planar shock wave. This is not a very interesting case, however, as all of the post-shock gradients are identically zero. The wedge flow problem is useful for testing the implementation of the Rankine-Hugoniot equations, grid generation, and other basic tools, but it provides no measure of the marching scheme's success.

A more interesting class of geometries for which exact solutions exist is the axisymmetric flow over a cone. The Taylor-Maccoll equation, a single, 2nd-order, ordinary differential equation, may be integrated numerically from the shock surface in toward the cone's surface, providing a solution for the flowfield between the shock and the cone that is exact within the bounds of the numerical integration. Details of the Taylor-Maccoll solution procedure can be found in Anderson[7]. For the results presented in this paper, 4th-order Runge-Kutta integration is used for the Taylor-Maccoll equation. Effects of grid resolution and limitations in the acceptable range of flowfields are discussed below.

For axisymmetric flowfields the accuracy and stability of the flowfield computation is independent of the number of points in the circumferential (η) or spanwise direction. However, if an accurate representation of the waverider's lower surface is desired, a reasonable number of points are needed in the spanwise direction, as each point corresponds to a streamline used in the surface definition as shown in fig. 7a.

The number of points in the streamwise and marching directions are necessarily coupled as can be seen in fig. 6. If more points are desired in the marching direction, then the number of points in the streamwise direction must be increased accordingly. The accuracy of the solution increases as the number of points in the marching direction is increased as shown in fig. 8, where the pressures predicted by the new marching code and by the Taylor-Maccoll equation are compared in the flowfield between the shock and the waverider's surface.

The algorithm is accurate and stable over a wide range of flow parameters. The range of applicability is bounded by an infinitely weak shock on one end, and a strong shock on the other end, where the post shock flow becomes subsonic. Approaching the strong shock limit does not present any real difficulty as long as the flow remains supersonic. However, when the weak shock limit is approached, solutions become difficult to obtain for two reasons. First, initial data must be given on a shock surface extending many body lengths downstream from the desired flow region in order to provide the needed downstream boundary conditions. As the shock strength approaches zero, the required length of the computational domain approaches infinity. Second, an unavoidable numerical error arises when the ξ-grid lines become parallel to the local velocity, as is often the case with weak shocks. The error is due to the contravariant velocity W showing up in the denominator of most of the right-hand-side terms of eqn. 12. If the velocity is parallel to the ξ-grid lines, then W is identically zero, and $\vec{q}_\zeta$ becomes indeterminate. This can be rationalized from a characteristics point of view. Recall that for rotational flow, streamlines form a third family of characteristics, and that if initial data is only given on a characteristic the solution may not be advanced off the characteristic.

Direct Simulations

Several of the present authors have assembled and validated a host of computational tools necessary for the direct simulation of inviscid flows about configurations with sharp leading edges such as waveriders[8]. These tools provide a means for validating the new design code results for which there are no exact solutions. Following are four test cases investigating different shock topologies with nonconstant shock strengths: case 1; an elliptic cone with $a/b = 0.75$, case 2; an elliptic cone with $a/b = 1.25$, case 3; a right circular cone at an angle of attack, and case 4; an axisymmetric surface with curvature in the streamwise direction.

In all cases the computational grids for the direct simulations had dimensions $41 \times 61 \times 31$ and were adapted three times (based on density gradients) during the convergence to more accurately capture the shock location. Solutions were allowed to converge until the l_2 norm dropped at least five orders, and the change in C_L, C_D, and L/D over 100 iterations was less than 10^{-5}. The resolution in the spanwise direction for the marching code, SCIEMAP (Supersonic Compressible Inverse Euler Marching Program), was 41 points, and the freestream Mach number in all cases was 4, as past experience has shown F3D to work better there than at higher Mach numbers.

Case 1 The shock surface in this case is a piece of an elliptic cone with $a/b = 0.75$ and a symmetry-plane shock angle of $\approx 26.5°$. The computational grid for SCIEMAP has 12 points in the marching direction. A freestream upper surface is used, and the resulting waverider is shown in fig. 9.

The values of C_L, C_D, and L/D predicted by SCIEMAP and F3D are given in tables 1 and 2, respectively. Note that the values of L/D differ by only 0.13%. Figure 10 plots the surface pressure predicted by both codes at 90% chord, showing excellent agreement.

Case 2 This shock surface is also a piece of an elliptic cone, and differs from case 1 only in the ratio of $a/b = 1.25$. The lower value of a/b used in case 1 results in a flowfield with a small pressure variation between the shock and surface (i.e., closer to planar flow), and the higher value of a/b used here results in a greater variation in pressure between the shock and surface, and is therefore a more difficult test. Fifteen points are used in the marching direction, and this case also includes a freestream upper surface, as pictured in fig. 11.

The ratio of L/D was computed within 0.12% of the value predicted by F3D. Surface pressures at 90% chord are compared in fig. 12 showing reasonable agreement between the two codes. The lower surface pressure distributions illustrate the accuracy of SCIEMAP.

Case 3 The shock surface in this case is a portion of a right circular cone with a cone angle of $\approx 22°$ and is at an angle of attack of $\approx 6°$. The marching grid has 16 points in the marching direction. The vehicle, shown in fig. 13, has an expanding upper surface that provides roughly 6% of the total lift.

The values of L/D in this case match to within 0.05%. Surface pressures at 90% chord are compared in fig. 14 and, as can be seen, agreement on the lower surface is quite good, and the agreement on the upper surface is within the expected tolerances for the method.

Case 4 The shock surface in this case is a portion of an axisymmetric surface where the arc of revolution is a piece of a parabola. The computational grid for SCIEMAP has 17 points in the marching direction. This configuration, shown in fig. 15, also includes an expanding upper surface with a more extreme expansion than case 3, providing roughly 17% of the total lift.

Predictions of L/D in this case match well with a 0.17% variation. Figure 16 compares surface pressures at 90% chord and, again, agreement on the lower surface is quite good, and on the upper surface the agreement is reasonable. The streamwise curvature of the shock in this case is quite small; however, the pressure distribution in the computed flowfield is remarkably different from that of a conical flowfield as can be seen in figs. 17a and 17b, respectively.

Table 1: Lift and drag results of SCIEMAP.

Case	C_L	C_D	L/D
1	0.2469	0.06609	3.7348
2	0.2403	0.06782	3.5438
3	0.2646	0.06966	3.7932
4	0.2697	0.06718	4.0149

Table 2: Lift and drag results of F3D.

Case	C_L	C_D	L/D
1	0.2444	0.06535	3.7396
2	0.2366	0.06669	3.5479
3	0.2605	0.06862	3.7963
4	0.2650	0.06590	4.0218

Conclusions

A new method for the design of waverider configurations with generalized shock geometries was presented. The new method provides increased flexibility over previous studies in the choice of shock shapes. The fundamental ill-posedness of the problem, which usually precludes the existence of bounded solutions, was suppressed by reformulating the problem in a curvilinear coordinate system, where the equations closely resemble a stable two-dimensional system by marching within the local osculating-plane.

Comparisons of results from the SCIEMAP algorithm with exact solutions for axisymmetric conical flow illustrate its accuracy over a broad range of parameters. Comparisons with direct Euler simulations using the F3D flow solver demonstrate its accuracy in computing complex flowfields, with more general shock geometries than any previous studies, and its application to waverider design has resulted in a new class of waverider topologies.

Acknowledgments

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Figure 1: Nonweiler's 'caret-wing' waverider.

Figure 2: Osculating Plane (OP).

Figure 3: Geometric layout of the shock surface.

Figure 4: Streamwise versus cross-stream marching.

Figure 5: Characteristic conoids in three-dimensions.

Figure 6: Symmetry-plane marching grid.

Figure 7a: Streamlines through the leading edge, and b). the waverider's spline-fit lower surface.

Figure 8: Pressure distribution between the shock and surface for axisymmetric conical flow.

Figure 9: Case 1: surface topology (- - - - shock profile).

Figure 10: Case1: surface pressure at 90% chord.

Figure 11: Case 2: surface topology (- - - - shock profile).

Figure 12: Case2: surface pressure at 90% chord.

Figure 13: Case 3: surface topology (- - - - shock profile).

Figure 14: Case3: surface pressure at 90% chord.

Figure 15: Case 4: surface topology (- - - - shock profile).

Figure 17a: Symmetry-plane pressure contours for a). nonconical flow (case 4) and b). conical flow (case 2).

Figure 16: Case4: surface pressure at 90% chord.